S282 Astronomy

Jeans mass

Study time: 90 minutes

Summary

In this spreadsheet activity you will create a formula to determine the Jeans mass of interstellar clouds. You will then use it to investigate the influence of the different cloud properties on the tendency of the cloud to contract.

You should have read Chapter 5 of *An Introduction to the Sun and Stars* before attempting this activity.

Learning outcomes

- Gain an awareness of the range of conditions under which a uniform spherical interstellar cloud is likely to collapse.
- Create a complex formula in a spreadsheet and use it to perform calculations.

Background to the activity

If the mass of a uniform spherical cloud exceeds the Jeans mass the force of gravitational attraction will overcome the opposing pressure due to the motion of the particles, and contraction will occur. This critical mass depends on the density, temperature and composition of the cloud, and is given by the expression:

$$M_\mathrm{J} = \frac{9}{4} \times \left(\frac{1}{2\pi n}\right)^{1/2} \times \frac{1}{m^2} \times \left(\frac{kT}{G}\right)^{3/2} \qquad (1)$$

(*An Introduction to the Sun and Stars*, Equation 5.1)

Although this equation provides a very simplified criterion for the collapse of an interstellar cloud it does provide a useful guide to the conditions under which that collapse is likely to occur. You will not consider here the complex aspects of cloud collapse such as inhomogeneities in properties of the cloud, rotation or magnetic fields. You will use the Jeans mass as a tool in investigate the conditions under which an ideal cloud is likely to collapse and to develop further your spreadsheet skills in use of formulae.

Part 1 The Jeans mass spreadsheet

The instructions given here assume that you will be using the StarOffice package that is supplied with the course and that you have completed some of the earlier spreadsheet activities. If you have not completed any other spreadsheet activities you will probably need to consult the *Using Spreadsheets* guide on the course website for instructions on some of the basic procedures not described in this activity, such as formatting cells.

If you are already familiar with using another spreadsheet package (such as Microsoft Excel) you may want to use that to carry out the activity. However, before starting, you should be aware that these notes only give instructions on how to manipulate the StarOffice spreadsheet.

Set up the spreadsheet

- If you have not already done so, start StarOffice. From the main StarOffice menu select File | New | Spreadsheet to create a new, blank sheet.

Don't forget to save your work regularly in your work folder. (You should already have set up such a folder for earlier activities. If not, create a work folder now.) From time to time make a backup copy of your work (using a different filename) in case you need to go back to an earlier stage.

As in the earlier activities, consider carefully the layout of your spreadsheet. In this activity you will construct a complex formula stage by stage, so it is vital that you include sufficient *labelling* and other information to allow any other user of the spreadsheet (or yourself if you return to the spreadsheet after some time) to understand what is going on. (You may wish to refer back to the formatting that you did in earlier activities as a reminder.)

The Jeans mass formula

In order to use the formula in a spreadsheet it is sensible to break it down into sections containing all the constants (which will be fixed in all calculations) and the variable quantities.

Question 1

For each symbol in the Jeans mass equation write down: its meaning, whether it is a constant or a variable, its value (if a constant) and its SI units.

- Enter a section in your spreadsheet with headings 'Constant' 'Symbol' 'Value' 'Unit' and insert the appropriate constants from the Jeans mass equation. To enter the value of π in the spreadsheet, use the function `=PI()`.
- Format the cells appropriately (scientific notation is required for most of the numbers).

Your spreadsheet should look like Figure 2 in Note 1 (see the 'Notes' section towards the back of this activity).

Question 2

Rearrange the Jeans mass equation so that all the constants are separated from the variables.

- Insert a new constant in your spreadsheet table (you can call it *A*), which is the combined constants you defined in the answer to Question 1. You can calculate the value of this constant using a calculator or you can enter it as a formula.

Question 3

What is the value of A? What are the units of A?

If you want to enter A as a formula you need to remember the basic rules for arithmetic. The order for arithmetic operations to be performed is:

^ (to the power of)

× and / (multiply and divide)

+ and – (add and subtract)

For example, when entering a number such as $5^{2/5}$ you will get the wrong answer if you enter `=5^2/5` since this represents $5^2/5$. The correct formula is `=5^(2/5)`. (Alternatively, you could write `=5^0.4`). If in doubt always use brackets to separate different parts of a formula and check your answers.

Question 4

What is the spreadsheet formula for the constant A?

Before you enter the formula for the Jeans mass equation you need to set up cells containing the variables.

- Below your list of constants make a new small table with similar headings for the variables in the Jeans mass equation. It is worth highlighting the 'Value' cells in a different colour since this is where you will want to enter some numbers. (See Figure 3 in Note 2.)

So far you have entered all numbers in SI units. It always makes sense to perform all calculations in SI units so that there is no confusion. However, you may want to enter numbers (or answers) in more convenient, non-SI units. Examples are molecular masses in units of the mass of a hydrogen atom (m_H), (or the Jeans mass in terms of solar mass ($M_\odot$)).

- Enter additional columns in your table showing the value and units of quantities in non-SI units.
- Enter formulae in the appropriate cells to convert them into SI units.

Question 5

What is the formula you would enter into the spreadsheet for the Jeans mass?

- Now enter the Jeans mass formula into your spreadsheet. (If you get an error message `Err.503` this may be because you have forgotten to enter values in the cells for the variables.)
- Format the cell to display the result in scientific notation. It is also a good idea to colour highlight the cell containing the result. You should use a colour that is distinguishable from one you used previously to highlight the 'Value' cells.
- Now enter an additional cell to convert the Jeans mass into units of solar mass.

(Your spreadsheet should now look like Figure 4 in Note 3.)

At this stage it is worth testing your results to ensure you have not made an error. With $T = 100$ K, $m = 1m_H$ (= 1.67×10^{-27} kg) and $n = 10^{10}$ m^{-3}, the Jeans mass is 3.03×10^{32} kg or $152.2M_{\odot}$.

Question 6

Use your spreadsheet to determine if the following clouds are likely to collapse:

(a) A cloud of mass $5M_{\odot}$ consisting entirely of neutral hydrogen with $T = 30$ K and $n = 10^{11}$ m^{-3}.

(b) A cloud with the same properties as in part (a) but consisting entirely of molecular hydrogen.

Part 2 Alternative versions of the Jeans mass spreadsheet

The spreadsheet you have created allows you to determine the Jeans mass for any set of cloud parameters. However, if you were asked the question: 'To what density would a molecular hydrogen cloud of mass $10M_{\odot}$ and temperature 20 K need to be compressed before it is likely to collapse?' it would require some trial and error to determine the answer using your spreadsheet.

It is not difficult to modify the spreadsheet to answer this question directly.

Set up a new worksheet

The new version can be entered in a different *worksheet*.

- At the bottom of the spreadsheet you will see a series of tabs marked Sheet1, Sheet2 and Sheet3. The current worksheet will be Sheet1.
- Select Format | Sheet | Rename... from the main menu and rename the sheet 'Jeans mass'. Select the tab Sheet2 and rename it 'Density'.

Since most of the Density worksheet will be the same as the Jeans mass worksheet, it is worthwhile copying the entire contents and editing them.

- Select the entire contents of the worksheet by clicking the square in the corner of the spreadsheet (shown in Figure 1). The worksheet will go dark to show the area selected. Select Edit | Copy from the main menu.

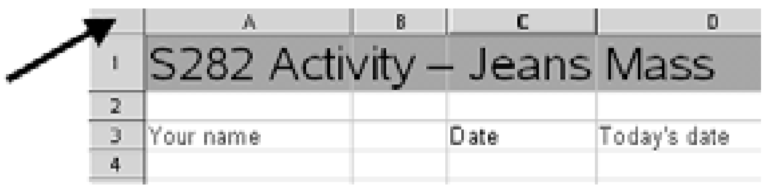


Figure 1 Select this cell to select the whole worksheet.

- Move to the Density worksheet and select the top left corner again. The worksheet will again go dark.
- Select Edit | Paste from the main menu and an exact copy of the Jeans mass worksheet will appear.

The Density worksheet

The only difference required in the new worksheet is that the Jeans mass is to be specified and the density calculated.

- In the *variables* section of your worksheet replace the row containing the density information with the Jeans mass (in SI units) as well as solar masses.
- Make sure you enter the appropriate conversion formula in the *value* cell to convert from solar masses to SI units. Enter an initial value of $1M_\odot$.
- Clear the row containing the Jeans mass formula by selecting the row (click on the row number to highlight the row) then select Edit | Delete contents from the main menu. Click OK in the window that appears.

You will now need to rearrange the Jeans mass equation to obtain the number density.

Question 7

What is the equation for the number density? How would you write this as a spreadsheet formula?

- Enter the formula for the number density in your spreadsheet. (See Figure 5 in Note 4 at the end of this activity.) Test your working using the same data as in the Jeans mass worksheet: $T = 100$ K, $m = 1m_{\mathrm{H}}$ ($= 1.67 \times 10^{-27}$ kg) and $M_{\mathrm{J}} = 3.03 \times 10^{32}$ kg or $152.2M_\odot$ gives $n = 10^{10}$ m^{-3}.

Question 8

Use your spreadsheet to determine the density above which the following clouds are likely to collapse:

(a) A cloud of mass $20M_\odot$ consisting entirely of neutral hydrogen with $T = 15$ K.

(b) A cloud with the same properties as in part (a) but consisting entirely of molecular hydrogen.

Explain why you would expect the result for part (b) to be lower than for part (a).

The Temperature worksheet

In this final part of the activity you will use your experience in setting up the Density worksheet to prepare a third worksheet to answer Question 9.

Note: if you have had difficulty in completing the earlier sections of this activity you may find it more instructive to revise your spreadsheet skills using the earlier spreadsheet activities and the *Using Spreadsheets* guide rather than attempting this part of the activity.

In order to answer Question 9 you will need to:

- Rearrange the Jeans mass equation in terms of the temperature.
- Create a new Temperature worksheet.
- Copy the contents of the Jeans mass worksheet into it.
- Modify the variables and formulae to derive the critical temperature for collapse.

(See Note 5 after you have attempted Question 9.)

Question 9

Use your spreadsheet to determine the temperature below which the following clouds are likely to collapse:

(a) A dense cloud of mass $3M_\odot$ and number density 10^{11} m^{-3} consisting entirely of molecular hydrogen.

(b) A diffuse cloud with the same mass and composition as the dense cloud in part (a) but with a number density of only 10^{7} m^{-3}.

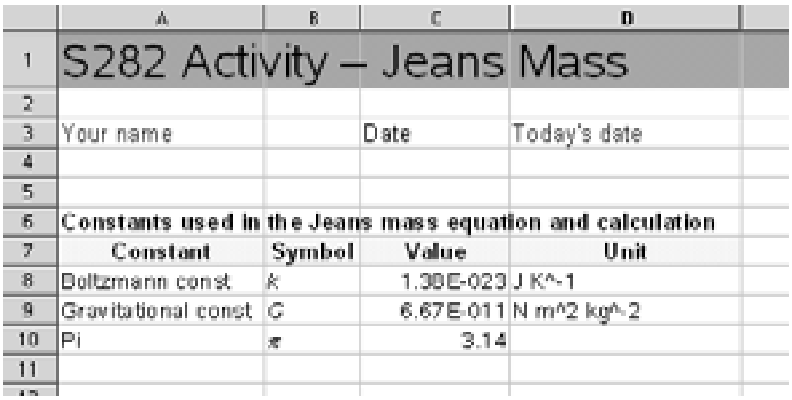
(c) A diffuse cloud with the same number density and composition as the diffuse cloud in part (b) but with a mass of $30M_\odot$.

Do the results match your expectations?

Notes

Note 1

When you have set up your Jeans mass spreadsheet it should look similar to Figure 2.



	A	B	C	D
1	S282 Activity – Jeans Mass			
2				
3	Your name		Date	Today's date
4				
5				
6	Constants used in the Jeans mass equation and calculation			
7	Constant	Symbol	Value	Unit
8	Boltzmann const	k	1.38E-023	J K^-1
9	Gravitational const	G	6.67E-011	N m^2 kg^-2
10	Pi	π	3.14	
11				

Figure 2 The spreadsheet after entering the constants used in the Jeans mass equation.

Note 2

After entering the variables used in the Jeans mass formula the spreadsheet should look something like Figure 3 (variables are entered in cells C15, C17 and E16).

	A	B	C	D	E	F
1	S282 Activity – Jeans Mass					
2						
3	Your name		Date	Today's date		
4						
5						
6	**Constants used in the Jeans mass equation and calculation**					
7	**Constant**	**Symbol**	**Value**	**Unit**		
8	Boltzmann const	*k*	1.38E-023	J K^-1		
9	Gravitational const	*G*	6.67E-011	N m^2 kg^-2		
10	Pi	π	3.14			
11	Combined constant	*A*	8.45E-020	m^-3/2 K^-3/2 kg^3		
12						
13	**Variables used in the Jeans mass equation and calculation**					
14	**Variable**	**Symbol**	**Value**	**Unit**	**Value**	**Non-SI unit**
15	Number density	*n*	1.00E+010	m^-3		
16	Gas particle mass	*m*	1.67E-027	kg	1	m_H
17	Temperature	*T*	100	K		
18						

Figure 3 The spreadsheet after entering the variables used in the Jeans mass equation.

Cell C16 contains the formula `=E16*1.67E-027`.

Note 3

After entering the Jeans mass formula the spreadsheet should look something like Figure 4 (the shaded cells indicate where variables are entered or key results displayed).

	A	B	C	D	E	F
1	S282 Activity – Jeans Mass					
2						
3	Your name		Date	Today's date		
4						
5						
6	**Constants used in the Jeans mass equation and calculation**					
7	**Constant**	**Symbol**	**Value**	**Unit**		
8	Boltzmann const	*k*	1.38E-023	J K^-1		
9	Gravitational const	*G*	6.67E-011	N m^2 kg^-2		
10	Pi	π	3.14			
11	Combined constant	A	8.45E-020	m^-3/2 K^-3/2 kg^3		
12						
13	**Variables used in the Jeans mass equation and calculation**					
14	**Variable**	**Symbol**	**Value**	**Unit**	**Value**	**Non-SI unit**
15	Number density	*n*	1.00E+010	m^-3		
16	Gas particle mass	*m*	1.67E-027	kg	1	m_H
17	Temperature	*T*	100	K		
18						
19	**Jeans mass**	*M_J*	3.03E+032	kg	152.2	M_sun
20						

Figure 4 The spreadsheet after entering the Jeans mass formula.

Cell E19 contains the formula `=C19/1.99E+30`.

Note 4

Your completed density worksheet should look similar to Figure 5.

	A	B	C	D	E	F
1	S282 Activity – Jeans Mass					
2						
3	Your name		Date	Today's date		
4						
5						
6	**Constants used in the Jeans mass equation and calculation**					
7	**Constant**	**Symbol**	**Value**	**Unit**		
8	Boltzmann const	*k*	1.38E-023	J K^-1		
9	Gravitational const	G	6.67E-011	N m^2 kg^-2		
10	Pi	π	3.14			
11	Combined constant	A	8.45E-020	m^-3/2 K^-3/2 kg^3		
12						
13	**Variables used in the Jeans mass equation and calculation**					
14	**Variable**	**Symbol**	**Value**	**Unit**	**Value**	**Non-SI unit**
15	Jeans mass	*M_J*	3.98E+031	kg	20	M_sun
16	Gas particle mass	*m*	3.34E-027	kg	2	m_H
17	Temperature	*T*	15	K		
18						
19	**Number density**	*n*	1.22E+008	m^-3		

Figure 5 The completed Density worksheet.

Note 5

Rearranging the form of the Jeans mass equation from Question 2:

$$M_J = \frac{9}{4(2\pi)^{1/2}}\left(\frac{k}{G}\right)^{3/2} \times \frac{T^{3/2}}{n^{1/2}m^2}$$

$$\text{gives } T^{3/2} = \frac{4(2\pi)^{1/2}}{9}\left(\frac{G}{k}\right)^{3/2} \times M_J m^2 n^{1/2}$$

$$\text{so } T = \left[\frac{4(2\pi)^{1/2}}{9}\left(\frac{G}{k}\right)^{3/2} \times M_J m^2 n^{1/2}\right]^{2/3} = \left[\left(\frac{1}{A}\right) \times M_J m^2 n^{1/2}\right]^{2/3}$$

The formula used in the temperature spreadsheet (Figure 6) will therefore be

```
=((1/C11)*C17*C16^2*C15^0.5)^(2/3)
```

	A	B	C	D	E	F
1	S282 Activity – Jeans Mass					
2						
3	Your name		Date	Today's date		
4						
5						
6	**Constants used in the Jeans mass equation and calculation**					
7	**Constant**	**Symbol**	**Value**	**Unit**		
8	Boltzmann const	k	1.38E-023	J K^-1		
9	Gravitational const	G	6.67E-011	N m^2 kg^-2		
10	Pi	π	3.14			
11	Combined constant	A	8.45E-020	m^-3/2 K^-3/2 kg^3		
12						
13	**Variables used in the Jeans mass equation and calculation**					
14	**Variable**	**Symbol**	**Value**	**Unit**	**Value**	**Non-SI unit**
15	Number density	n	1.00E+010	m^-3		
16	Gas particle mass	m	1.67E-027	kg	1	m_H
17	Jeans mass	M_J	3.03E+032	kg	152.2	M_sun
18						
19	**Temperature**	T	100	K		

Figure 6 The completed Temperature worksheet.

Answers to questions

Question 1

Table 1 Variables and constants in the Jeans mass equation.

Symbol	Meaning	Variable/constant	Value	SI Units
M_J	Jeans mass	variable		kg
π	the number pi	constant	3.142…	
n	particle number density	variable		m^{-3}
m	mass of the 'average' gas particle in the cloud	variable		kg
k	Boltzmann constant	constant	1.38×10^{-23}	$J\ K^{-1}$
T	temperature	variable		K
G	gravitational constant	constant	6.67×10^{-11}	$N\ m^2\ kg^{-2}$

Question 2

The Jeans mass equation

$$M_J = \frac{9}{4} \times \left(\frac{1}{2\pi n}\right)^{1/2} \times \frac{1}{m^2} \times \left(\frac{kT}{G}\right)^{3/2} \tag{1}$$

can be rewritten with all the constant terms separated, as:

$$M_J = \frac{9}{4(2\pi)^{1/2}} \left(\frac{k}{G}\right)^{3/2} \times \frac{T^{3/2}}{n^{1/2} m^2}$$

Question 3

The value of A is

$$\left(\frac{9}{4\times(2\pi)^{1/2}}\right)\times\left(\frac{1.38\times10^{-23}}{6.67\times10^{-11}}\right)^{3/2} = 8.45\times10^{-20}$$

The units of A are obtained from the units of k and G (all other terms have no units).

The units are

$$\left(\frac{\mathrm{J\,K^{-1}}}{\mathrm{N\,m^2\,kg^{-2}}}\right)^{3/2}$$

which can be written

$$\left(\frac{\mathrm{N\,m\,K^{-1}}}{\mathrm{N\,m^2\,kg^{-2}}}\right)^{3/2} = (\mathrm{m^{-1}\,K^{-1}\,kg^2})^{3/2}$$

$$= \mathrm{m^{-3/2}\,K^{-3/2}\,kg^3}$$

Question 4

One way to write the formula for A is

```
=(9*(1.38E-23/6.67E-11)^1.5)/(4*(2*PI())^0.5)
```

There are many alternatives, but they should all give the correct answer!

A simpler way is to use the cell references for the values of k, G and π, which would be written

```
=(9*(C8/C9)^1.5)/(4*(2*C10)^0.5)
```

for the spreadsheet in Figure 2.

Question 5

The formula for the Jeans mass in the spreadsheet is

```
=C11*C17^1.5/(C15^0.5*C16^2)
```

Question 6

(a) Entering $m = 1m_{\mathrm{H}}$ (= 1.67×10^{-27} kg) for neutral hydrogen, $T = 30$ K and $n = 10^{10}$ m^{-3} gives a Jeans mass of $7.9M_\odot$. The cloud mass is smaller than the Jeans mass so it will not collapse.

(b) Entering $m = 2m_{\mathrm{H}}$ (= 3.34×10^{-27} kg) for molecular hydrogen, $T = 30$ K and $n = 10^{10}$ m^{-3} gives a Jeans mass of $2.0M_\odot$. The cloud mass now exceeds the Jeans mass so it is likely to collapse.

Question 7

Rearranging the form of the Jeans mass equation from Question 2:

$$M_{\mathrm{J}} = \frac{9}{4(2\pi)^{1/2}}\left(\frac{k}{G}\right)^{3/2}\times\frac{T^{3/2}}{n^{1/2}m^2}$$

gives $n^{1/2} = \frac{9}{4(2\pi)^{1/2}}\left(\frac{k}{G}\right)^{3/2} \times \frac{T^{3/2}}{M_J m^2}$

so $n = \left[\frac{9}{4(2\pi)^{1/2}}\left(\frac{k}{G}\right)^{3/2} \times \frac{T^{3/2}}{M_J m^2}\right]^2 = \left[A \times \frac{T^{3/2}}{M_J m^2}\right]^2$

The formula will therefore be

```
=(C11*C17^1.5/(C15*C16^2))^2
```

Question 8

(a) The critical density for collapse is $1.95 \times 10^9\ \mathrm{m}^{-3}$. If the cloud is compressed to a density greater than this value it will collapse.

(b) The critical density for collapse is $1.22 \times 10^8\ \mathrm{m}^{-3}$.

The critical density does not need to be so high for a molecular hydrogen cloud. The gravitational forces, which must overcome the outward pressure in the gas for collapse to occur, depend on the total mass of the cloud, the masses of individual particles and their separations (as defined by the number density). The total mass of each cloud is the same, but the mass of each particle in the molecular hydrogen cloud is higher than in the neutral hydrogen cloud. A particle at the edge of the molecular hydrogen cloud does not therefore need to be so close to the centre of mass of the cloud to feel the same gravitational force as a particle at the edge of the neutral hydrogen cloud. The molecular hydrogen cloud can therefore be larger, i.e. have a lower number density than the neutral hydrogen cloud, for collapse to occur.

Question 9

(a) The critical temperature below which the cloud is likely to collapse is 40 K.

(b) The critical temperature is 1.8 K.

(c) The critical temperature is 9 K.

The critical temperature for the low density diffuse cloud in (b) is lower than for the higher density cloud in (a) with similar properties. The two clouds have the same total mass and the same particle mass, but the mean separations of the particles in the diffuse cloud are greater than in the dense cloud. This means that the diffuse cloud in (b) must be larger than the dense cloud in (a). A particle at the edge of the diffuse cloud will therefore be further from the centre of mass of the rest of the cloud and the gravitational force on it will be lower. In order for collapse to occur the gas pressure (as determined by the motions of the particles, and hence the temperature) must therefore also be lower.

If the mass of the diffuse cloud is raised but the number density remains unchanged then the cloud size must also increase. The gravitational force on a particle at the edge of the cloud increases because the effect of increase in mass more than compensates for the increase in distance from the centre of mass. (The gravitational force on the particle is proportional to M/R^2. Since R is proportional to $M^{1/3}$, then the gravitational force is proportional to $M/(M^{1/3})^2$ or $M^{1/3}$. So, as mass increases the gravitational force increases.) As the cloud mass is increased the gravitational forces are therefore increased and the temperature does not need to be so low for collapse to occur.

(Note: this analysis is very simplistic and ignores all the other factors which may govern the collapse of interstellar clouds.)